Abstract—A fully adaptive radar framework has been proposed in recent publications, and this paper will implement the framework in an adaptive update rate application for a tracking radar. A cost function is developed to balance the radar resource usage with the track error. The method is illustrated with a simulated example to show how the radar could operate in a simple scenario. Using a cognitive radar experimental testbed, a similar scenario to the simulation is tested to show how a cognitive radar acts in a real world environment. The method enables the radar to operate using minimum resources when the target acts predictably. Conversely, the resource usage increases when the target maneuvers or track error increases.

I. INTRODUCTION

In a cognitive radar (CR), the radar forms a perception of the environment by measuring it with electromagnetic radiation, and the perception is then used as the basis for decision making to support a goal or objective. The decision could be to use a different waveform, or to adapt in any of the degrees of freedom the radar has. Two strategies for this approach can be found in [1] and [2].

CR adapts to the environment through feedback, normally from the receiver to the transmitter which allow the radar to operate in environments that would be considered diverse and changing for a conventional radar. Ultimately, CR will have the ability to learn, such that when similar situations are encountered in the future, the optimal solution can be reached rapidly.

Using attention, the CR can focus its perception, and hence the most important parts of it’s resources towards the most important parts of the environment. The radar has finite resources available and often multiple objectives to achieve, such as surveillance, tracking, weather monitoring, weapon guidance etc. Most commonly, the radar time line is regarded as the primary resource, however, other resources such as bandwidth, transmit power and processing power could be manged. Focusing attention on critical parts of the radar’s perception, the limited resources can be shared between multiple functions. CR has the potential to enable radars to use resources more optimally, and that can therefore adapt to changing environments.

This work will show how the track update interval can be adapted using a CR framework for development of a cost function that balances the use of radar resources against track error. In comparison to the work in [3] where the pulse repetition frequency (PRF) is adapted, this work focuses solely on the update interval and keeps all other parameters fixed.

The algorithm developed, based on solving the cost function as an optimization problem, was implemented in a simulator and on a CR experimental testbed. The final part of this work show results from both simulation and experimental data.

II. COGNITIVE RADAR RESEARCH

A CR uses ideas and principles derived in cognitive psychology that emulate functions of the brain. CR is a relatively new field of radar research, initially outlined in the paper of Haykin [1] and book by Guerci [4]. Haykin’s description of CR is based on the work of neuropsychologist Fuster [5], who describes a mechanism he calls the perception-action cycle. In the cycle, actions are selected based upon the perception. The actions will result in changes to the perception over time leading to selection of new actions, and hence the process continues indefinitely. The cycle is based on reaching some goal or end-state, and actions are made such that the goal is reached in an as optimal fashion as possible with as low cost as possible. Haykin [6] describes the perception-action cycle in the brain as a feedback system between the perceptor, which can be seen as the receiver part of a radar system, and the actuator, which can be seen as the transmitter part.

Although Haykin’s description of CR is largely based on the work of Fuster and cognitive psychology, there has been a great deal of work published on knowledge aided radar systems [4], [7] in a CR context. This work has been more focused on using knowledge together with adaptive radar and waveform diversity to optimize radar performance in applications such as Space Time Adaptive Processing (STAP). A large body of work in the waveform diversity and adaptive radar community [8], [9] has enabled the development of CR using many of the techniques for waveform selection and adaptation.

Research into biologically inspired methods is also considered to support the idea of a CR. [10], [11] show how methods originating in the biological understanding of bats can be used for guidance and control of a radar equipped robot in a maze. This research helps to demonstrate that a CR can be responsible for more than just adapting its waveform. The actions it selects can be steering commands for the platform that carries it. Under these situations, it is the change in
platform position that results in the perception change of the perception-action cycle.

Further work on CR is demonstrated in [12] where anticipation is used to find some optimal distribution of the tasks the radar must accomplish when there are known obstructions to these tasks, such as the need to dedicate a large amount of the radar time line to a specific objective like SAR imaging.

Bell et al. [13] showed a general CR framework, which can be instantiated for different tasks. Examples are given for a single target tracking problem and for resource management in a network of sensors.

III. FULLY ADAPTIVE RADAR FRAMEWORK FOR UPDATE-INTERVAL CONTROL

An important parameter for resource allocation in a multi-target tracking radar is the update-interval of each track. When the update-interval is short, too many resources may be used maintaining the track, while long intervals can save resource, but could result in large errors, or even a broken track. Work has been done on update-interval selection in phased array tracking radars [14]–[17] where a steady state approximate solution to the predicted error should not exceed the beam width in azimuth and elevation. Van Keuk introduced the criterion in [18], and used it in an example where the update-interval is calculated based on a steady state solution to the Kalman filter model, described by Singer in [19].

A fully adaptive radar (FAR) framework for CR was developed in [13] for general tracking systems to simplify the development of CR systems. The framework introduces a feedback model between the processor and transmitter for optimal waveform selection based on minimizing the inverse of the Fisher information matrix (FIM), weighted against the processor cost function. Bell investigated the implementation of the framework on an software defined radar (SDR) where the PRF was the adapted parameter [3]. In this case, the number of pulses integrated was kept constant, and hence the update-interval is short, too much resources may be used maintaining the track, while long intervals can save resource, but could result in large errors, or even a broken track. Work has been done on update-interval selection in phased array tracking radars [14]–[17] where a steady state approximate solution to the predicted error should not exceed the beam width in azimuth and elevation. Van Keuk introduced the criterion in [18], and used it in an example where the update-interval is calculated based on a steady state solution to the Kalman filter model, described by Singer in [19].

B. FAR framework and tracker model

The FAR framework was implemented using the Singer tracker model as a state-space representation of a moving target and yields

\[ x_{k+1} = \phi_k(T_k)x_k + w_k \]  
\[ z_k = Hx_k + v_k \]

where \( w_k \sim N(0, Q_k(T_k)) \), \( v_k \sim N(0, R_k(\Delta r_k, \Delta v_k)) \) and \( T_k \) is the update interval. The motion model \( Q_k \) is given in [19] and the measurement accuracy \( R_k \) is given using the accuracy model found in [20, pp. 689-699]. It states that the lower bound for accuracy is

\[ \sigma_R \geq \frac{\Delta R}{\sqrt{SNR}} \]  
\[ \sigma_V \geq \frac{\sqrt{3} c}{\pi} \frac{\Delta v}{f_0 \sqrt{SNR}} \]

The state space consists of range, velocity, acceleration, azimuth and SNR. The upper block of the transition matrix shown [19], and the lower block is a simple identity matrix stating that the change is only due to white Gaussian process noise.

The Kalman filter recursion shown in [19] is used for motion and information update, where the predicted covariance matrix from the information update is equal to the predicted information matrix (PIM) [13] for a Gaussian density. The predicted conditional Cramér-Rao lower bound (PC-CRLB) is defined as the inverse of the predicted conditional Bayesian information matrix (PC-BIM) where the PC-BIM is equal to the sum of PIM and expected value of the FIM. The expected value of the FIM is defined as

\[ J_k(\theta_k|Z_{k-1}; \Theta_{k-1}) = E_k\{J_x(x; \theta_k)\} \]  
\[ J_x(x; \theta_k) = -E\{\nabla_x[ln f(z_k|x_k; \theta_k)]\} \]

The pdf \( f(z_k|x_k; \theta_k) \) is a multivariate Gaussian distribution with covariance \( R_k \) and zero mean. The FIM can be shown to be

\[ J_w(x_k; \theta_k) = E\{H^T R_k^{-1} H\} \]

The FIM is not a function of \( x_k, z_k \) or \( \theta_k \), and the expected FIM is therefore equal to the FIM. The PC-BIM is therefore given as

\[ B_k^*(\theta_k|Z_{k-1}; \Theta_{k-1}) = \Sigma_k(\theta_k)^{-1} + H^T R_k^{-1} H \]

where \( \Sigma_k \) is the predicted posterior covariance calculated from the Kalman filter. The PC-BIM has the property [3]

\[ R_{\text{PC}}^*(\theta_k|Z_{k-1}; \Theta_{k-1}) \geq tr\{B_k^*(\theta_k|Z_{k-1}; \Theta_{k-1})^{-1}\} \]

A cost function for the CR should balance the resources a track update require and the PC-BIM. The resource requirement for a track update could be defined as an inverse relationship
of the update interval, to emulate the increased cost when the update interval is smaller. Hence, the cost function for this FAR system could be defined as a balance between the PC-BIM and an inverse function of the update interval.

\[ L_{C,\theta}(\theta_k|Z_{k-1};\Theta_{k-1}) = R_C(\theta_k|Z_{k-1};\Theta_{k-1}) + \frac{\theta_0}{\theta_k} \]  

(10)

The constant factor \( B_0 \) is a weighting factor, where a large trace value would emphasize radar resource usage. The factor \( \theta_0 \) is a weighting factor, where a small value emphasize the size of the PC-BIM. The factors could be pulled together in the weighting factor \( K = B_0\theta_0 \). For each iteration, the radar would then solve the minimization problem

\[ \theta_k = \arg \min_{\theta} \left[ \frac{1}{\theta_k} \right] \left[ (\Sigma_k(\theta)^{-1} + H^T R_k^{-1} H)^{-1} \right] + \frac{K}{\theta} \]  

(11)

The cost function from the minimization problem is balancing between radar resource usage and the size of the PC-BIM to find a compromise.

IV. RESULTS

A. Simulation results

A simulator has been built in Matlab to test applications using the FAR framework. The radar parameters were selected to resemble the experimental radar system built at Norwegian defense research establishment (FFI) [21] for testing of CR applications. A simulated target moving away from the radar on a straight line, offset from the antenna boresight, with constant velocity is depicted in figure 1, with range on top, velocity in the middle and signal to noise ratio (SNR) at the bottom.

For the radar to sustain track quality when the target maneuvers or the distance to the radar increases, a natural solution would be to decrease the revisit interval of the track. Using the balancing cost function shown in (11), the solution of the minimization problem is to reduce the update interval for the next iteration. Figure 2 shows the target tracker covariance, for range and velocity in the top two subplots, and update interval on the bottom subplot. The covariances increases as the target range increase due to the coupling of SNR and range/velocity accuracy given in (3) and (4), and the Kalman update dependent on SNR and accuracy. The trend of the update interval is decreasing as the covariance increase and the SNR decreases. The solution of the minimization problem ensures a balance between resource usage and track quality given by the cost function.

After approximately 50 seconds, the radar start to loose detections as the SNR fluctuates below the detection threshold. The covariance increases since there has been no track update, and hence the update interval decreases to the lowest threshold. The method therefore decreases the possibility of a lost track.
Figure 3. Experimental data of target position with FAR

since it increases the update interval until a stable track is regained.

B. Experimental results

A similar scenario to the simulation was tested on the CR testbed developed at FFI [21]. Target range, velocity and SNR are shown in figure 3, where a target is detected and tracked as it moves away from the radar at an approximately constant rate. The SNR does not behave according to an inverse range to the fourth power rule, as might be expected when moving away from the radar, and the reason for this was attributed to propagation mechanisms in the scene and the target’s motion through the radar beam pattern. The target moves into the center of the beam as it moves away from the radar, and hence the loss of SNR is not as great as expected.

Figure 4 shows how the target track covariance and update interval evolved over time. There is a connection between increased covariance and shorter update intervals. When the target accelerates or decelerates, the covariance increases and the update interval decreases accordingly. The target moves away from the radar, and the SNR decreases as time evolves. Just as for changes in acceleration, the update interval decreases as the SNR decreases because a falling SNR also tends to increase the track covariance. Being able to compensate for these two effects should enable the target tracker to maintain a track longer during maneuvers and at increased detection ranges.

Figure 4. Experimental data of target track parameters with FAR

V. CONCLUSION

The FAR framework has been demonstrated as capable of controlling the track update rate for CR undertaking single target tracking. A cost function for this application was developed. A simulation and experiment using a CR testbed showed that the update intervals were selected in a balanced manner weighing radar resource usage against track covariance. Decreasing the update interval in situations where the covariance increases enabled the radar system to maintain track of maneuvering targets and distant targets when the SNR decreased. The balancing between track error and radar resource usage enables the radar to operate in a more efficient manner, using less resources when the target acts in a predictive way, and intensifying the resource usage when the target is less predictive. Using a cost function and solving an optimization problem instead of a set of heuristic rules for update interval selection, enables the radar to handle a larger variety of situations.

This work focused on single target tracking, but implementing several solvers of the optimization problem for each track in a multi-target tracking system, can allow the methodology to be transferred to a multi-target tracking system. Each solver will produce an optimum update interval for its respective track, and can then be used in the track update policy based on the radar system architecture.

Further work should look into how the FAR solution is...
compared to other adaptive tracking radar methods for update interval control. More complex scenarios should be investigated to look into how the radar operates when exposed to less predictive targets. Introducing more features from CR such as learning and intelligence should be considered to improve the flexibility and performance.

REFERENCES